Structural Analysis of Bunching Cavity for MEBT Section of PXIE Test Facility

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This note summarizes studies made during the cavity design stage and having as a goal understanding the impact of static and dynamic loads applied to elements of the cavity at different stages of fabrication, transportation, installation, testing, and use.

The material used for the cavity fabrication is oxygen-free high conductivity electronic copper Cu-10100. The mass of the cavity is ~75 kg.

Relevant properties of this material are summarized in the next table.

Density	8940.6 kg/m^3
Young's modulus	117 GPa
Poison ratio	0.33
Yield strength	69 MPa

A 3-D artistic rendering of the cavity is shown in Fig. 1 with the main dimensions in Fig. 2.

Fig. 1. Artistic view of the bunching cavity.

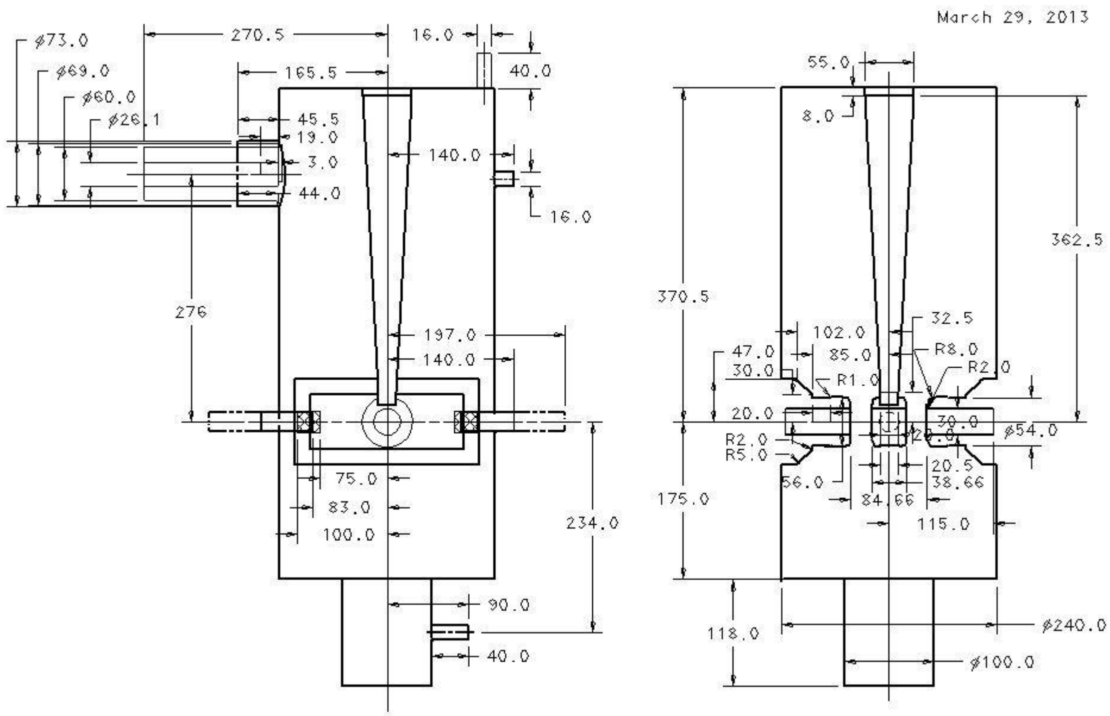

Fig. 2. Main dimensions of the RF volume of the bunching cavity

The thickness of the cavity cylinder is ~9 mm in the lower part, and it is ~12 mm in the area of the power coupler to ensure reliable connection of the coupler port by brazing.

I. Atmospheric pressure

The impact of atmospheric pressure on the cavity frequency change is described here in pure mechanical sense; dielectric properties of air were not taken into account. Fig. 3 shows maps of stress and deformations in the cavity after the air is pumped out. The model was constrained on the top part of the cylinder thick wall ring in all directions. Differential pressure of 10^5 Pa (about 1 bar – atmospheric pressure) was applied on the outer surfaces of the cylinder. The maximum displacement of 0.01 mm was recorded with the equivalent stress less than 8 MPa, which is well below the yield strength. Using deformed structure as input for the RF eigenvalue problem, corresponding shift of the resonant frequency can be found; it does not exceed 1 kHz. This frequency shift can be compared with sensitivity of the frequency to other factors listed in the table below.

Cavity temperature	-3 kHz/degC
Length of the Stem Cylinder	-350 kHz/mm
Position of a tuner relative to the axis	+10 kHz/mm
Diameter of the cavity	-580 kHz/mm
Permittivity of air inside the cavity	-40 kHz

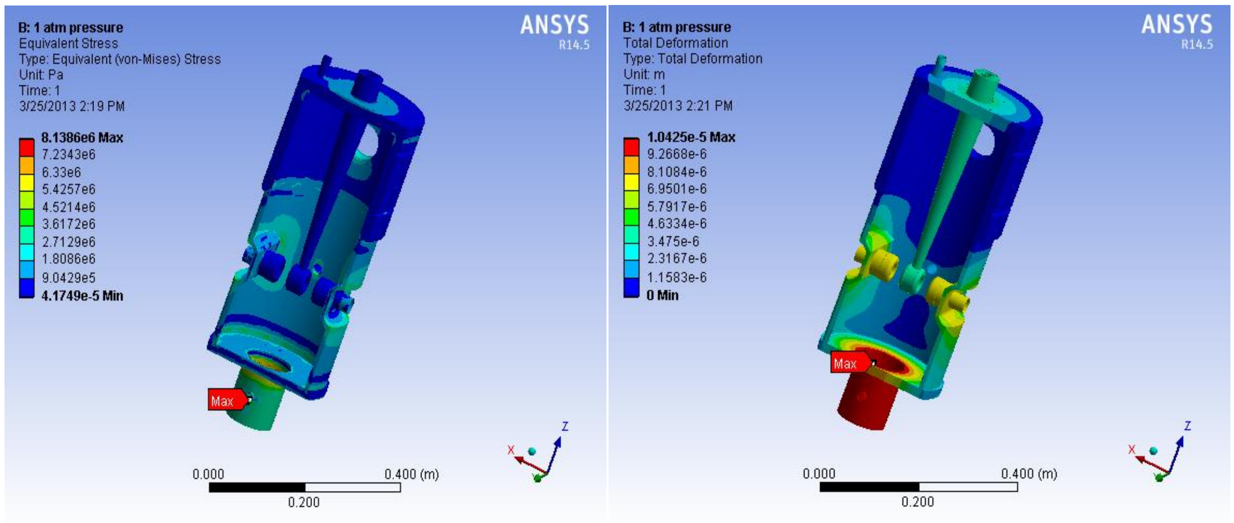

Fig. 3. Impact of atmospheric pressure

II. Hydrostatic Pressure

As water is used in the central stem of the cavity, one needs to make sure that hydraulic pressure (up to 20 Bar in accordance with the FRS) does not result in an unwanted structural problem. Fig. 4 shows distribution of the stress and deformation due to the hydrostatic pressure at the bottom of the water channel.

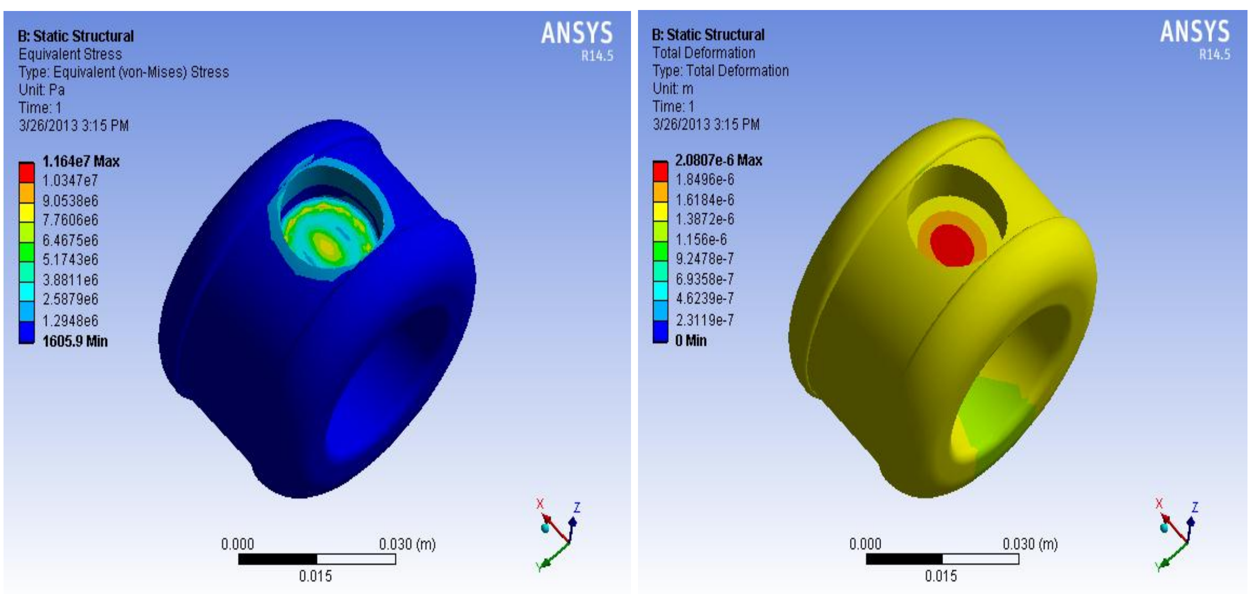

Fig. 4. Stress and deformation at the bottom of the water channel due to hydrostatic pressure.

Dynamic pressure due to the water flow pattern in the cooling channel can be evaluated assuming 5 GPM water flow. This flow results in additional 0.15 bar of dynamic pressure, which is much lower than the hydrostatic pressure.

III. Static loads.

Understanding the impact of static loads applied to flanges of the cavity can prevent from exceeding the limits of plastic deformation when the cavity is installed in the beam line and when the coupler and the tuners are added to the assembly. To obtain the maps of deformation and stresses sample unit forces (1 N) and momentums (1 N·m) were applied to corresponding elements (flanges of the cavity) in the directions of each main axis, and corresponding reaction was recorded. Table below provides information about allowable limits, based on the elasticity limit of 69 MPa.

Location & load direction	Unit load stress on copper (Pa)	Unit load deformation (mm)	Max. allowed force (N)	Allowed force moment (N-m)	Deformation at max. allowed force (mm)
Beamline Compression	4541	1.02E-05	15195		1.58E-02
Beamline sideload - Y	4683	7.46E-06	14734	440	1.12E-02
Beamline sideload - Z	5155	8.99E-06	13385	400	1.23E-02
Coupler sideload - Y	2737	2.37E-06	25210		6.09E-03
Coupler sideload - Z	3279	2.51E-06	21043		5.38E-03
Tuner sideload - Y	13946	1.98E-05	4948		9.99E-03
Tuner sideload - Z	31364	6.72E-05	2200		1.51E-02

The cavity seems quite rigid structurally; nevertheless some reasonable (and modest) measures should to be considered when the structure is installed in the beam line. Examples of these measures can be using bellows in the beam line and stress relief fixture for the RF feed cable. Tuner ports are the most sensitive to the applied force; although one would not expect any problems during installation of the tuners, they must be protected from accidental impact when other activities take place in the isles on both sides of the beam line.

IV. Using lifting fixture

To understand whether using lifting fixture can result in an irreversible deformation of the cavity, the model was constrained on top lifting hooks. Standard earth gravity was applied to the structure within the model. The maximum equivalent stress of 1.79 MPa was recorded on the cavity surface with the maximum displacement of ~2 μm (Fig. 5).

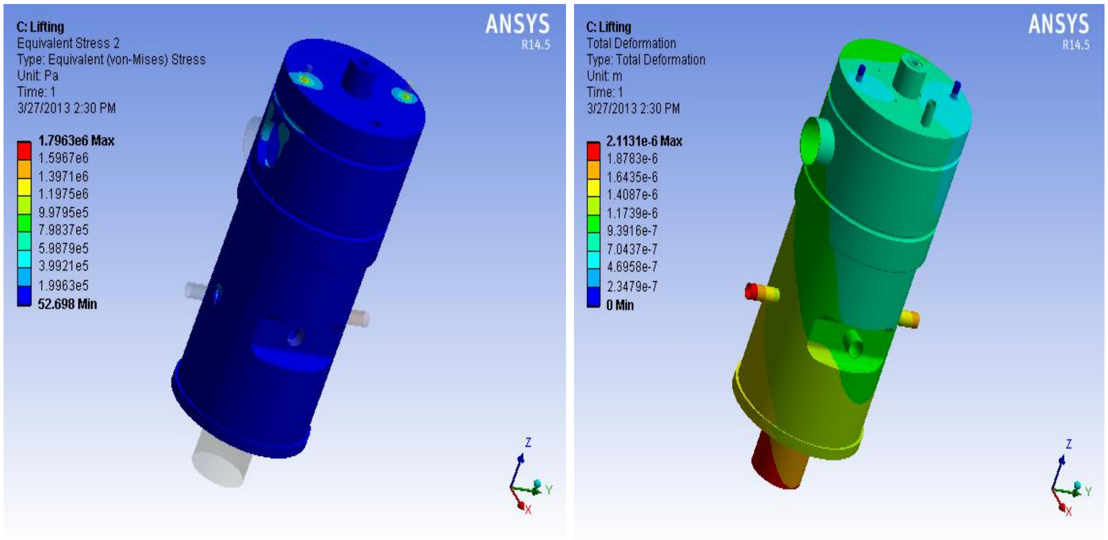

Fig. 5. Stress and deformation when the lifting fixture is used.

V. Impact of cavity acceleration

During transportation, cavity will inevitably experience multiple cycles of static, pulsed, and periodic inertial forces. These forces develop during loading/unloading (and can be of a shock type) and during transportation (ground and air) due to both vibration and aperiodic acceleration. The impact of the acceleration was investigated by applying a body force equivalent to $1 \cdot g$ acceleration in the direction perpendicular to the axis of the cavity. Comparing resultant stress with the yield limit of copper one can get a limit for an acceptable acceleration.

At the specified conditions, the maximum stress in the material is ~500,000 Pa, and the displacement is ~10 µm. The limit of plastic deformation is reached when the acceleration reaches $\sim 100 \cdot g$. The end of the stem deflection at this acceleration is ~1 mm.

So, the support during cavity transportation will have to limit possible static acceleration to a safe limit of $\sim 20 \cdot g$.

Although this allowed acceleration value seems sufficiently high, certain modes of mechanical vibrations during handling, transportation, or after installation in the beam line, if not constrained, can lead to dangerously high oscillation amplitude.

VI. Mechanical oscillations of the central stem

The amplitude of the stem vibrations can only be evaluated if the quality factor of the vibration system is known. Let’s try to understand what vibration amplitude we should expect at the end of the stem if the base of the cavity is oscillating with frequency Ω and amplitude δ in the plane of the top (or bottom) end.

Motion of the end of the stem in the horizontal direction (as in Fig. 2) can be expressed in form of the next differential equation:

$$\partial^2 x/\partial t^2 + \alpha\cdot\partial x/\partial t + \omega_0^{\;2}\cdot(x-x_0(t)) = 0$$

Here we assume that the amplitude of oscillation (A) is much larger than the shift of the suspension point x_0, so that we can neglect corresponding changes in the velocity and acceleration. We will assume then $x_0(t) = \delta\cdot\sin(\Omega\cdot t)$. Then we have an oscillator with the friction dumping $\boldsymbol{\alpha}$ and forced excitation:

$$\partial^2 x/\partial t^2 + \alpha\cdot\partial x/\partial t + \omega_0^{\;2}\cdot(x) = \omega_0^{\;2}\cdot\delta\cdot\sin(\Omega\cdot t)$$

Solution is found in the next form: $x = A\cdot\sin(\Omega\cdot t+\varphi)$, which leads to the next equations for A and φ:

$$A = \omega_0^{\;2}\cdot\delta\cdot\{(\omega_0^{\;2}-\Omega^2)^2 + \alpha^2\cdot\Omega^2)^{-1/2}$$

$$tg(\varphi) = -\alpha\cdot\Omega/(\omega_0^{\;2}-\Omega^2)$$

Two cases need to be analyzed: off the resonance without damping and in the resonance.

1. $|\omega_0^{\;2}-\Omega^2| \gg \alpha\cdot\Omega$

In this case the oscillation frequency is relatively far from the resonant condition, and the amplitude of oscillation

$$A = \omega_0^{\;2}\cdot\delta / |\omega_0^{\;2}-\Omega^2|$$

2. $\omega_0 = \Omega$ – resonance

In this case, the amplitude of oscillation is fully defined by the dumping in the oscillating system:

$$A = \omega_0\,\delta/\alpha$$

Quality factor is generally defined as a ratio of the resonant frequency to the full frequency width of the resonance curve, where the oscillation amplitude changes by √2 times. This translates into the next relationship between the quality factor and the loss (dumping) coefficient α:

$$\alpha = 2\Delta\omega = \omega_0/Q$$

Then, the expression for the amplitude at resonance can be re-written as

$$A = \delta\cdot Q$$

Another parameter that is often used while dealing with the dumped oscillations is an isotropic loss factor η_s:

$$\eta_s = \alpha/\omega_0 = 1/Q$$

If the base of the central stem is oscillating with the frequency that is close to the resonance and the quality factor of the oscillator is high, amplitude of oscillation at the end of the stem can reach values that can introduce irreversible deformation, which changes RF properties of the cavity. Theoretical limit of the quality factor of a mechanical system in the elastic region can be found if to take into account thermo-elastic effect, which is one of mechanisms of the dissipation of the oscillator's energy through transfer of the heat generated in a bending structure due to thermodynamic heating. A simplified theoretical approach [1] uses the thermal relaxation factor

$$\tau = \rho \cdot C_p \cdot h^2/(\pi^2 \cdot k)$$

to calculate the quality factor of the oscillating beam using the next expression:

$$1/Q = E \cdot \alpha^2 \cdot T/(\rho \cdot C_p) \cdot \omega\tau/(1+(\omega\tau)^2).$$

In these equations, ρ is the density of the material (copper) - ρ = 8700 kg/m^3, C_p is specific heat: C_p = 385 J/kg-K, h is the equivalent transverse dimension of the oscillating beam: h = 40 mm was accepted for this exercise, k is the heat transfer coefficient: k = 400 W/(m·K), E is the elasticity (Young's) modulus: E = 110 GPa, α is the thermal expansion coefficient: $\alpha = 17 \cdot 10^{-6}$ 1/K, and T is the temperature of the oscillator: T = 300 K was accepted here.

The thermal relaxation factor calculated for the bunching cavity stem geometry $\tau \approx 0.75$ s. Using expected oscillation frequency f ≈ 230 Hz for the lowest mechanical mode, the quality factor related to the thermo-elastic effect is found to be ~400,000. As this quality factor seems very high, verification was made using the thermo-elastic branch of MEMS module of COMSOL [2]. Fig. 6 shows the deflection pattern of the lowest mode of oscillation and gives the complex eigenfrequency of the mode: $f \approx 227.8 + j \cdot 3.4 \cdot 10^{-4}$.

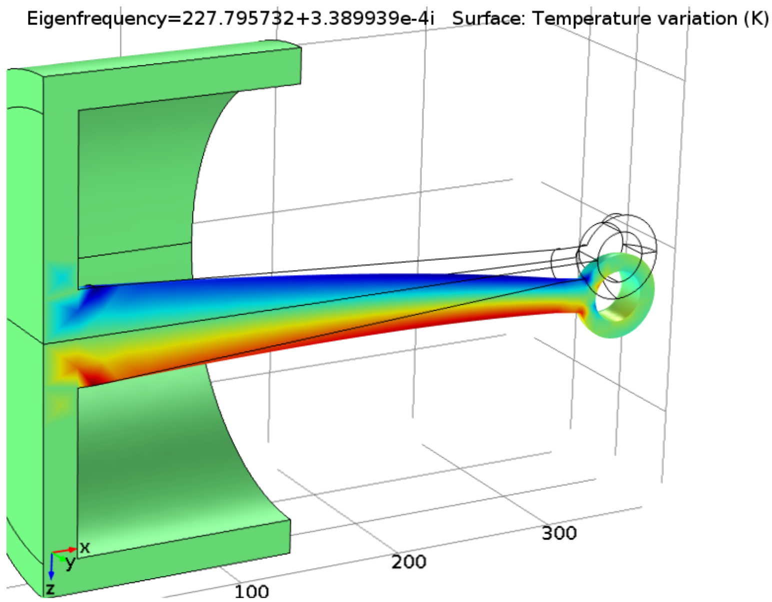

Fig. 6. Basic oscillation mode with thermo-elastic heating

Quality factor of this oscillation mode can be found as Q = Re(f)/[2·Im(f)] ≈ 340,000, which is consistent with the simplified theoretical evaluation.

This high quality factor only represents the upper limit; other factors (like viscosity or air or nonlinear structural properties) will make the quality factor of the oscillations much smaller. Nevertheless even in the case when Q = 10,000, the expected amplitude of the stem oscillations at the resonant frequency (with the 1 μm amplitude forced oscillation of the base) is ~10 mm, which significantly exceeds the 1 mm safe limit of the oscillation amplitude.

Direct modeling of forced oscillations near resonance for the configuration in Fig. 6 is an agreement with the predictions of the theoretical model.

The next two resonant frequencies of the system are 692 Hz and 1045 Hz; Fig. 7 shows corresponding spatial modes. Quality factors of these modes are also very high.

To avoid excessive stress in the stem that would exceed the plasticity limit, measures for dumping mechanical oscillations with the frequencies above ~100 Hz must be found and used during any stage of transportation of the device. Limiting the amplitude of any possible vibrations will definitely help.

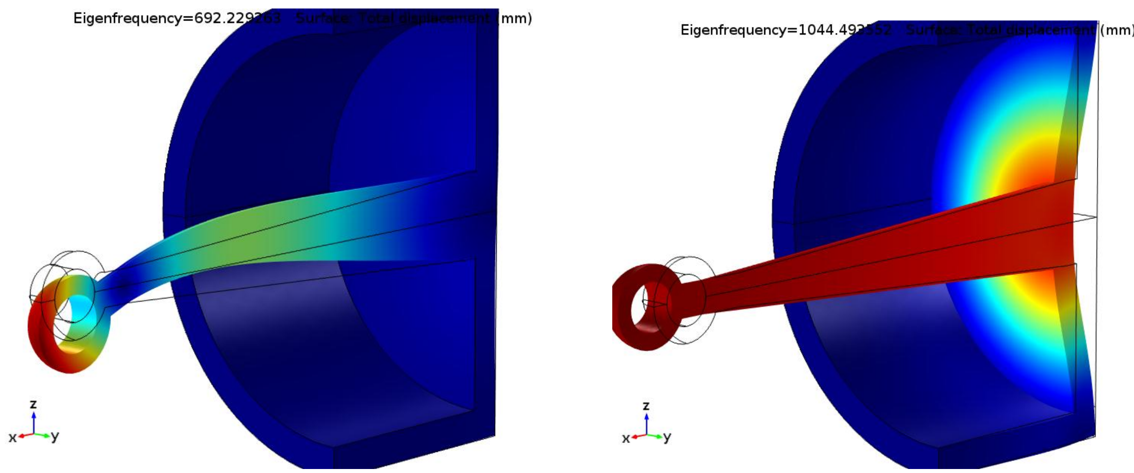

Fig. 7. The second and the third lowest resonant modes of the stem oscillation.

VII. Conclusion

Analysis of mechanical properties of the bunching cavity was made. Limits of static loads applied to different ports of the cavity during installation in the beam line, or to its surface (e.g. atmospheric or hydraulic pressure), are defined. Although the limits seem safe, careful handling of the cavity must be exercised not to exceed the safe limit of the loads accidentally. The most dangerous mechanical factor for the cavity is vibration of the central stem. During transportation, reasonable measures must be taken to limit movement of the stem and to introduce some damping in order to reduce quality factor of possible mechanical oscillations. Preferred position of the cavity during transportation is vertical; this position will mitigate the effect of the (most dangerous and highly expected) vertical vibrations during transportation. Design of a crate for the cavity transportation and of an oscillation-dumping fixture to use on the assembly must be considered as important part of the cavity fabrication and delivery process.

References:

1. C. Zener, “General Theory of Thermoelastic Internal Friction”, Physical Review, v. 53, pp. 230-235, 1938
2. COMSOL Multiphysics Engineering Simulation Software: http://www.comsol.com/products/multiphysics/